# Sensor Placement for Detection of Cracks in Structures Exhibiting Nonlinear Dynamics


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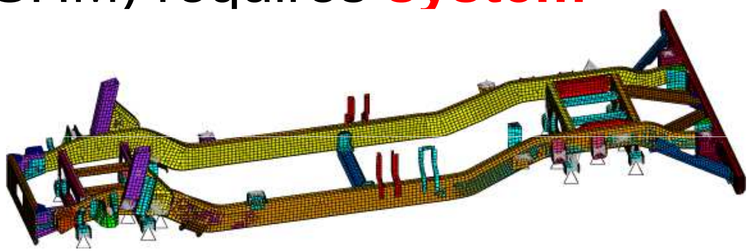
# Modeling and damage detection for complex structures

## Challenges:

- Component-level **damage** affects system-level dynamics
- Fast re-analysis is needed to **reduce computational cost** of large-scale finite element models
- Cracks create **nonlinear dynamics** (much harder to tackle)
- Structural health monitoring (SHM) requires **system information**: sensors



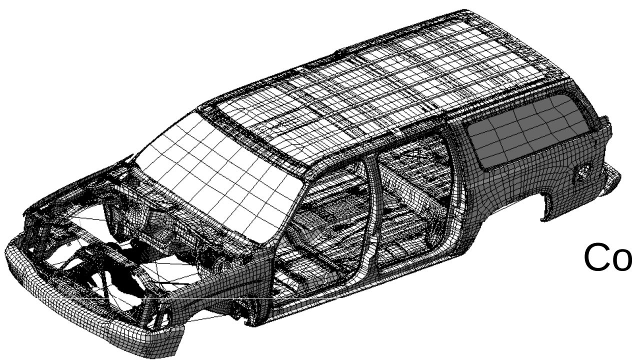
*Vehicle frame model (developed by Prof. Hulbert, Dr. Ma, Dr. Hahn of the Univ. of Michigan)*

## Approach:

- Apply component-based methods to assemble system-level reduced-order models (ROMs) of damaged structures
- Employ linear approximations of nonlinear (cracked) structural dynamics
- Combine above into sensor placement / measurement point selection algorithm

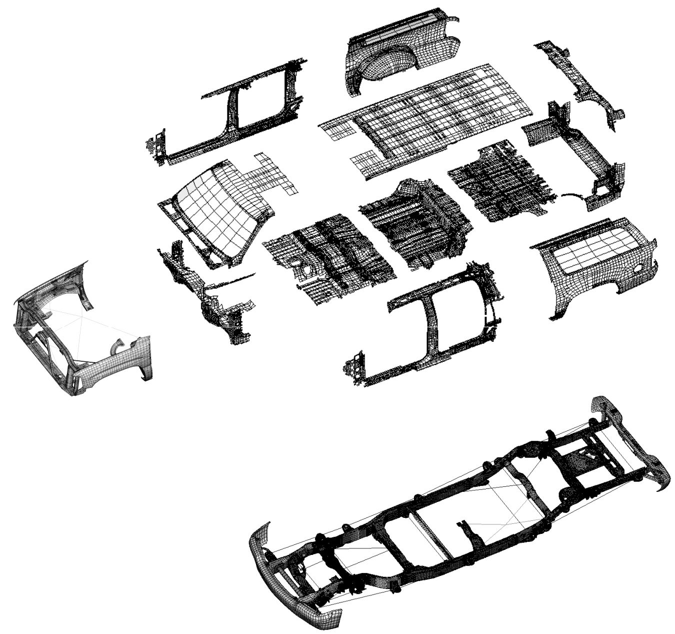
# Reduced Order Models: Overview

- Dynamic analysis of **invariant** complex structures
  - Projection by lower modes of the large-scale eigenvalue problem



Divide and Conquer

Component Mode Synthesis (CMS)

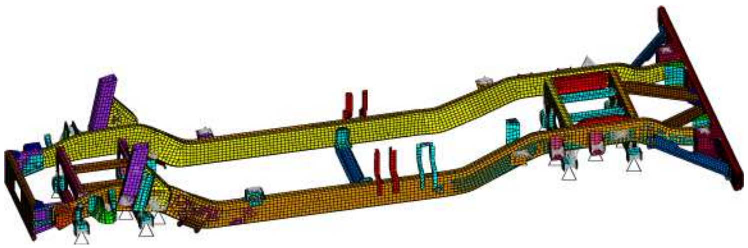


Takes more than a day only for the dynamic structural analysis

- Dynamic analysis of **damaged** complex structures
  - Projection by **proper basis** of the large-scale eigenvalue problem
  - Proper basis can be defined for **each damage type**: **cracks, dents and other structural variations** of complex structures

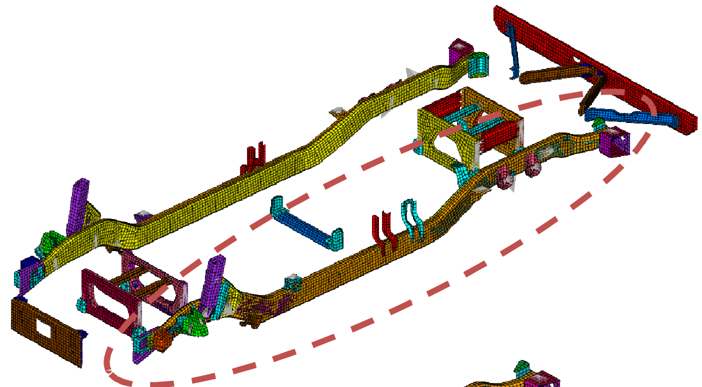
# Reduced Order Models: Substructuring

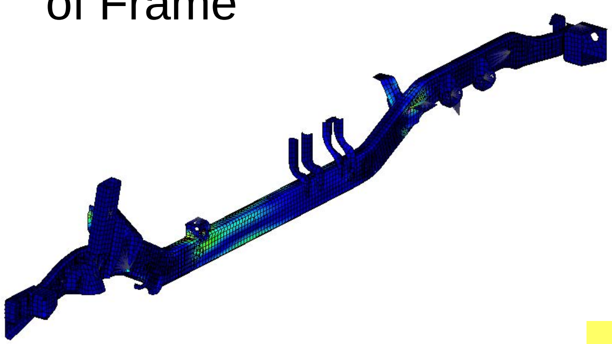
- Assemble ROMs of system (e.g., frame) from finite element **analyses of components and subcomponents**
- Efficiently **predict vibration, loading, stress in critical regions**



Finite Element Model of Frame

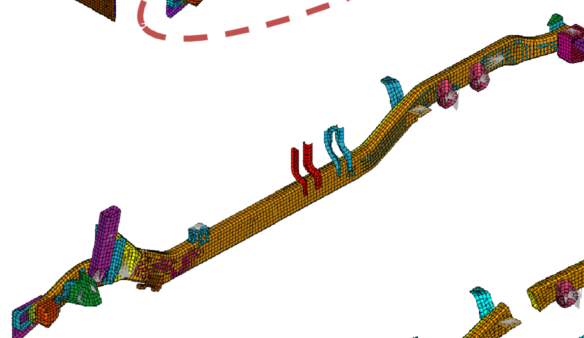
System Level: Vehicle Frame



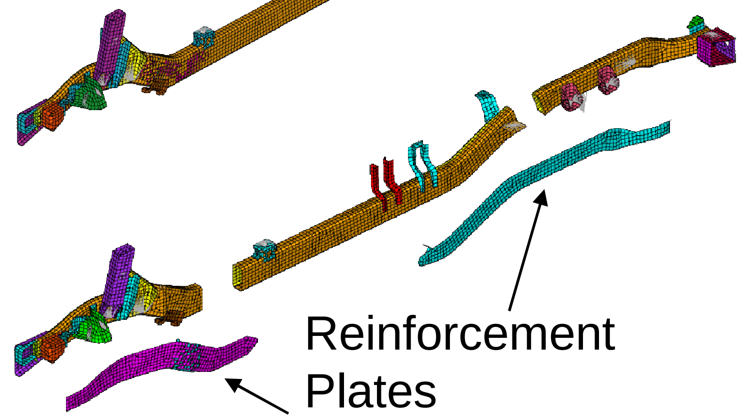


Dynamic stress for component mode (left rail)

Component Level: Left Rail

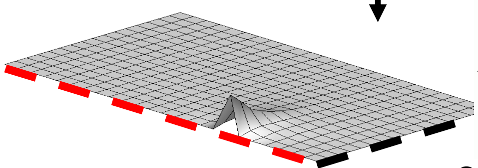


Subcomponent Level: Rail Sections, Reinforcement Plates

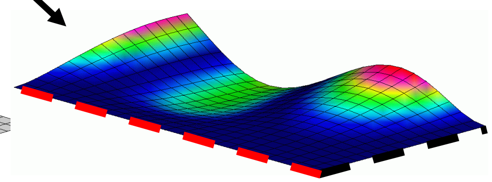


# Reduced Order Models: CB-CMS

$$\begin{bmatrix} u_A \\ u_O \end{bmatrix}_{(i)} = \begin{bmatrix} I & 0 \\ \Psi_C & \Phi_N \end{bmatrix}_{(i)} \begin{bmatrix} u_A \\ p_O \end{bmatrix}_{(i)}$$

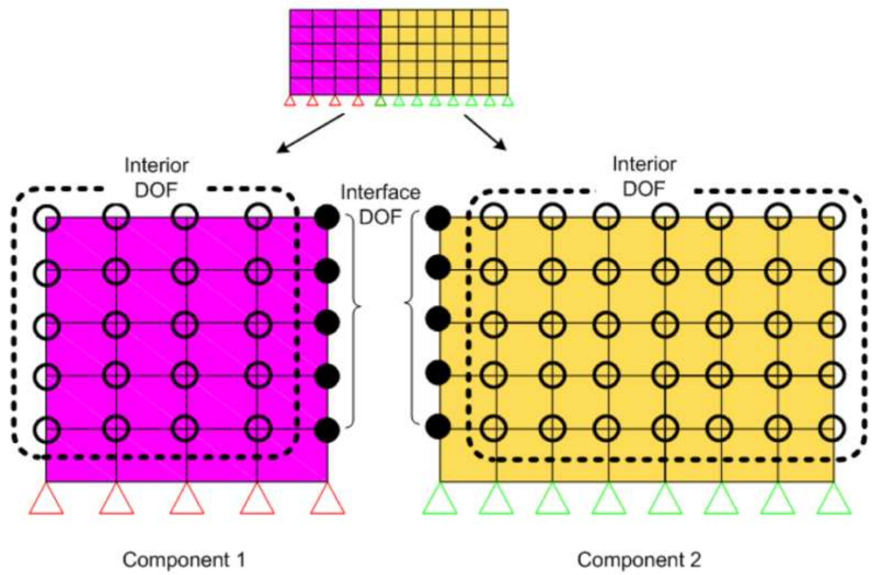


Static modes: as many as interface DOF



Component 1

Selected fixed-interface normal nodes




- *i* th component mass and stiffness matrix and force vectors

$$\mathbf{M}_i^{CBCMS} = \begin{bmatrix} \mathbf{m}_i^{\;C} & \mathbf{m}_i^{\;CN} \\ \mathbf{m}_i^{\;NC} & \mathbf{m}_i^{\;N} \end{bmatrix} \qquad \mathbf{K}_i^{CBCMS} = \begin{bmatrix} \mathbf{k}_i^{\;C} & 0 \\ 0 & \mathbf{k}_i^{\;N} \end{bmatrix} \qquad \mathbf{F}_i^{CBCMS} = \begin{Bmatrix} \mathbf{f}_i^{\;C} \\ \mathbf{f}_i^{\;N} \end{Bmatrix}$$

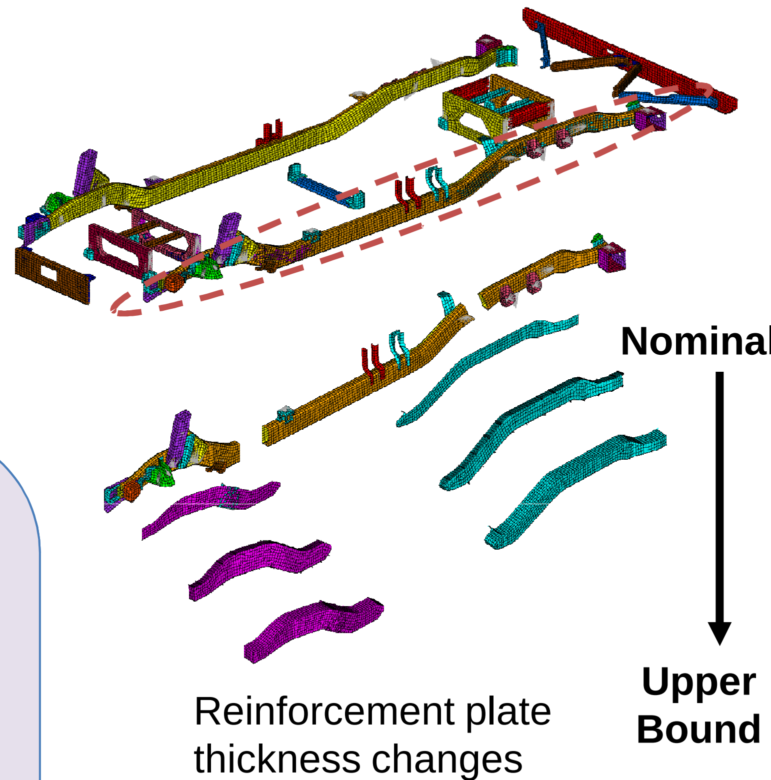
- Superscript C : Constraint part

- Superscript N : Internal part

- Subscript *i* : *i*th component

# Reduced Order Models: Parametric Models (PROMs)

- Enable **fast re-analysis**
- Subcomponent dynamics evaluated at **sampled parameter values**
- System-level **response expressed as function of parameter changes**




**- Global PROM (Parametric Reduced Order Models)**

- Balmès: Collected eigenvectors at sampled points in the parameter space
  Problem: Overhead computational cost to get the modal matrix to project the FE model

**- CMB-PROM (Component Mode Basis PROM)**

- Zhang (2005): Collect fixed interface normal modes and global interface mode and project the FE model.
  Problem: Global analysis not substructural analysis

**- Component PROM**

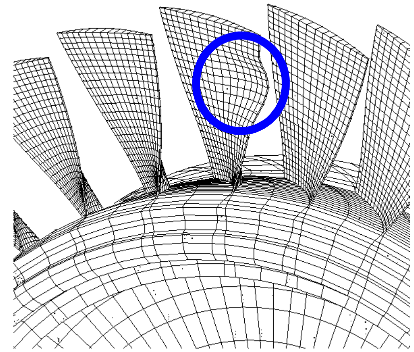
- Park (2008): Developed PROM for substructural analysis
  Problem: a single design component is tackled

**Multi-component PROM (MC-PROM)**

# Reduced Order Models: Static Mode Compensation

## Geometrical variations of the structure (dents)

- Lim (2004): used SMC for vibration of turbomachinery bladed disks for geometrical mistuning using SMC



*j*th healthy system mode

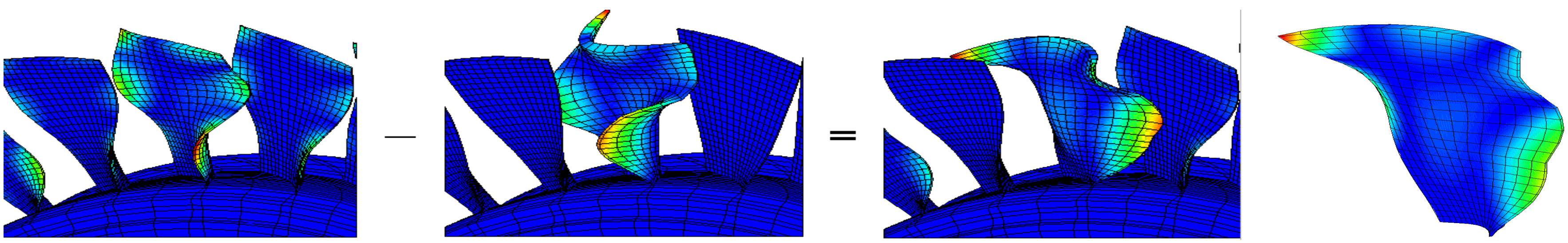
*j*th Static Mode

Effect of damage

by assuming external force

$$\mathbf{f}_j = (\mathbf{K}^{\delta} - \omega_j^{S^2} \mathbf{M}^{\delta}) \phi_j^S$$

$$\phi_j^S - \mathbf{K}^{m-1} \mathbf{f}_j$$



Normal mode (33,100.23 Hz) − Quasi-static mode (centering frequency: 34,000 Hz) = Basis shape

Damaged blade mode (34,563 Hz)

**Global structure analysis**

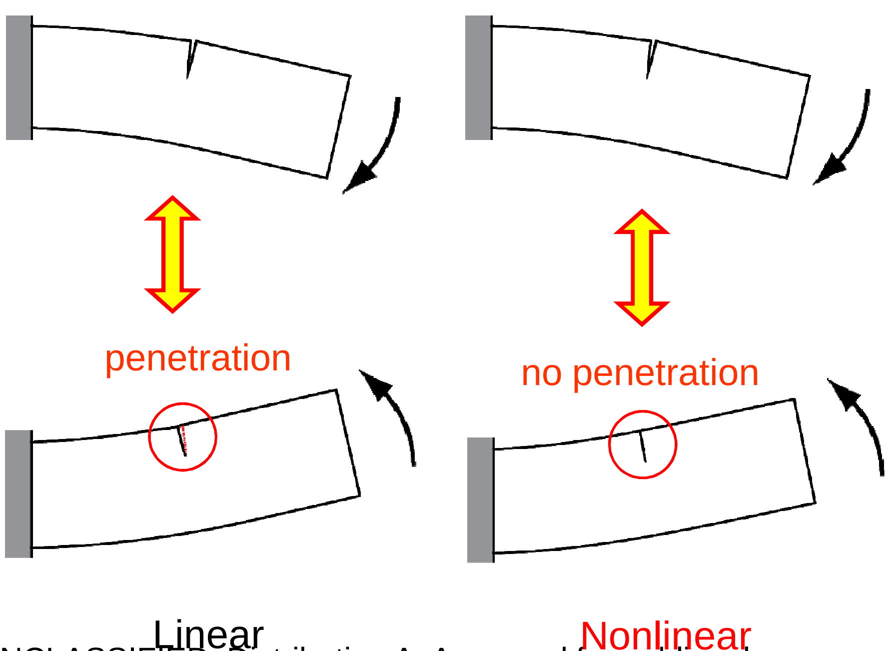
**not component-level analysis**

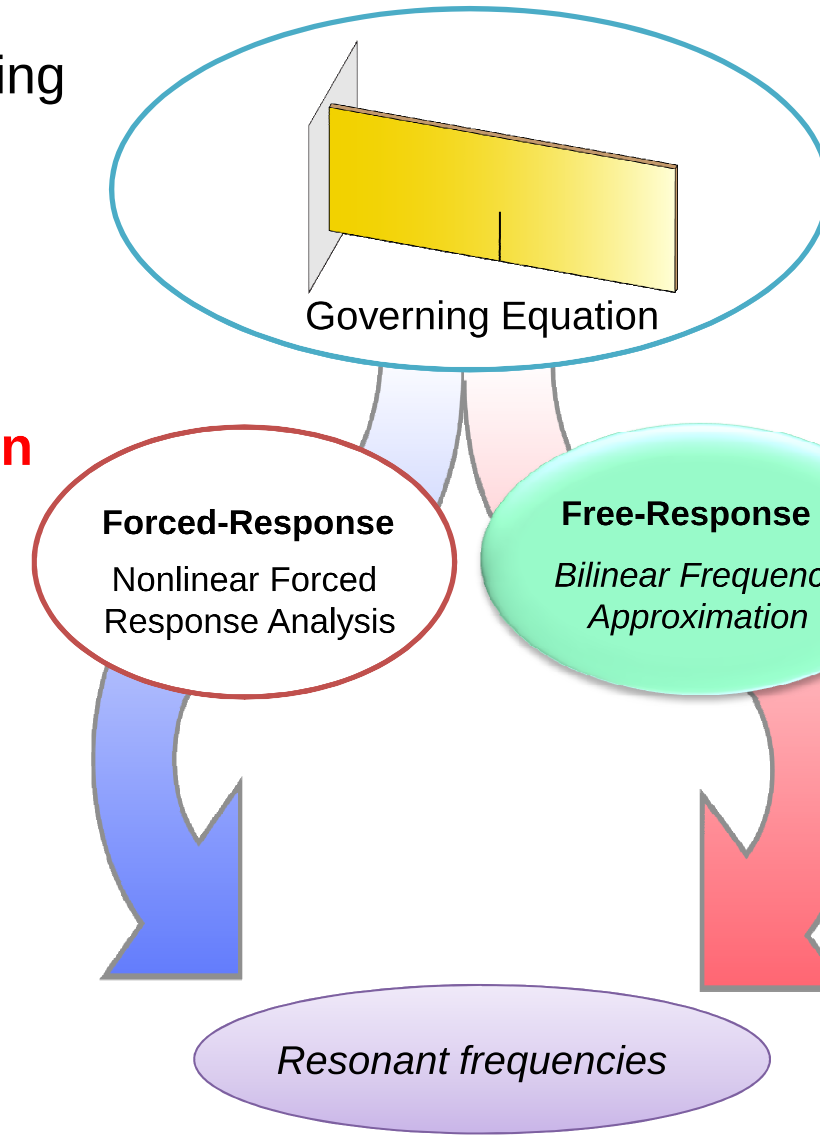
**Component Mode Synthesis with Static Mode Compensation (SMC-CMS)**

# Reduced Order Models: Nonlinear Dynamics: Cracks

## Cracks in the structure

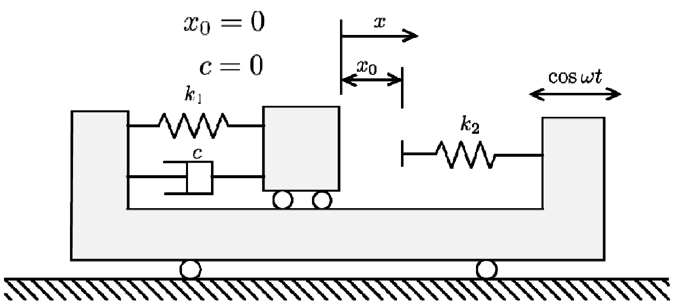
- Crack surfaces open and close during vibration: **nonlinear vibration**
- Hybrid Frequency / Time Domain method (Poudou 2003)
- Bilinear Frequency Approximation (Shaw 1983): **no mode information**



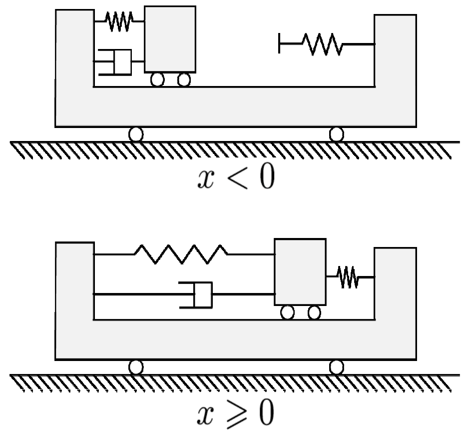



# Reduced Order Models: Bilinear Frequency Approximation

Exact for nonlinear vibration frequency of a piecewise linear oscillator




Single DOF, piecewise linear oscillator

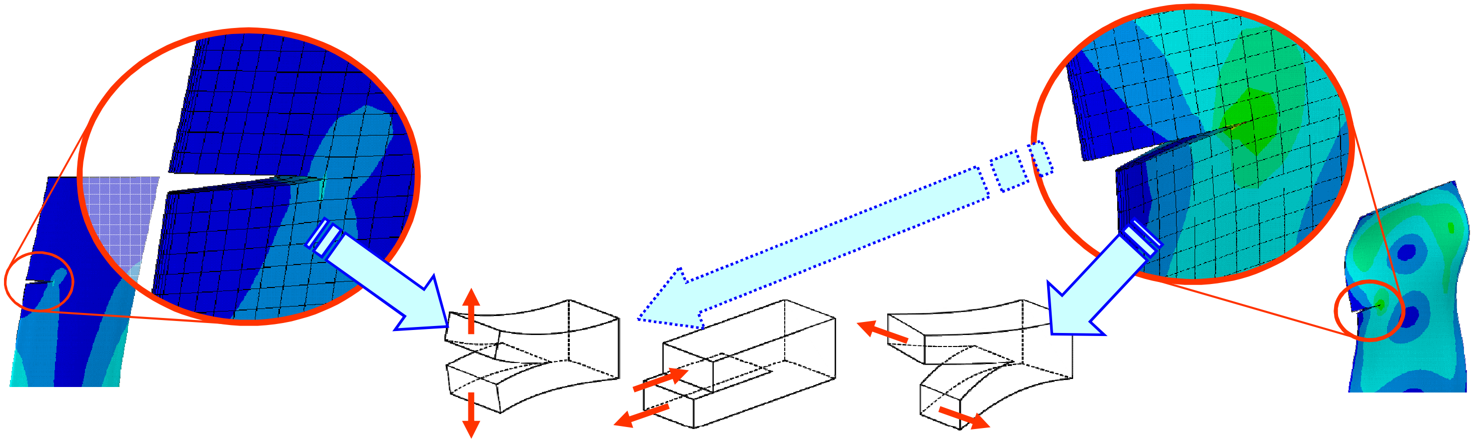



Bilinear Frequency

$$\omega_b = \frac{2\omega_1\omega_2}{\omega_1 + \omega_2}$$

- Bilinear frequency approximation (BFA) for **multiple DOF** (Chati et al., 1997)
- BFA using **general 3D finite element model** (Saito et al., 2009)

# Reduced Order Models: Bilinear Mode Approximation



Opening mode (I) Sliding mode (II) Tearing mode (III)

Crack modes

- Manage **boundary conditions on the crack**: open and closed cases
- Crack open: open boundary condition: **DOF on crack surface are free**
- Crack closed: sliding boundary condition: **free sliding inside crack surface**

- Mode approximation: **shape of vibration** is a linear combination of mode shapes for open and closed crack cases (dominant coherent structures)

**Bilinear Mode Approximation (BMA)**

# Reduced Order Models: Framework

## Analysis Framework

- Divide the global structure into substructures with or without damage
- Apply Craig-Bampton CMS (CB-CMS) for substructures which do not have any damage or variability
- Apply MC-PROM for the substructure with model variations (e.g. uncertainties)
- Apply BFA for cracked structure analysis

Assemble substructures for M&S of system-level response under various damage locations and crack lengths, uncertainties, design changes

## Core technologies

- CB-CMS
- Multi-Component PROM
- SMC-CMS
- Bilinear Frequency and Mode Approximation

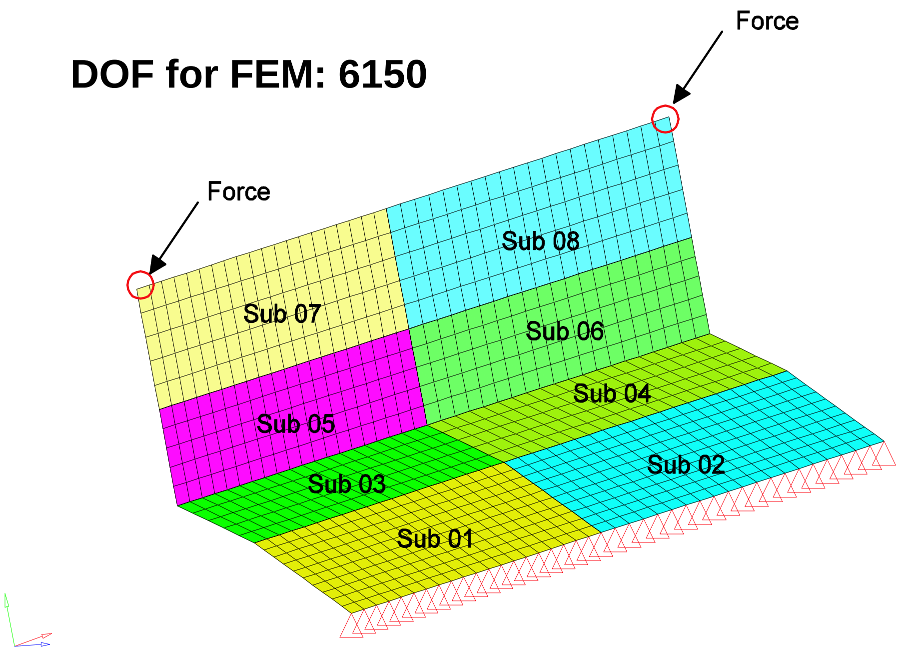
**Efficient framework for damage detection and for structural predictions**

# Example: L-Shape Plate : Dents and Thickness Variations

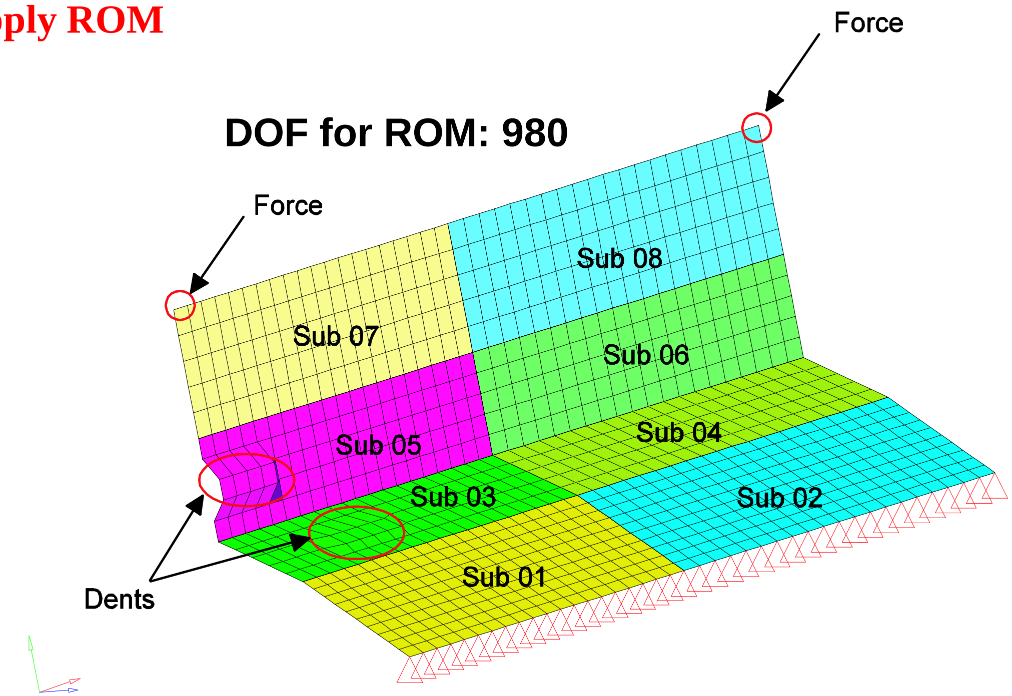
$$-\omega^2 \mathbf{M}_{FEM} u_{FEM} + (1 + j\gamma) \mathbf{K}_{FEM} u_{FEM} = \mathbf{F}_{FEM} \quad \Rightarrow \quad -\omega^2 \mathbf{M}_{ROM} q_{ROM} + (1 + j\gamma) \mathbf{K}_{ROM} q_{ROM} = \mathbf{F}_{ROM}$$

Apply ROM

**DOF for FEM: 6150**




**DOF for ROM: 980**




## Thickness variations

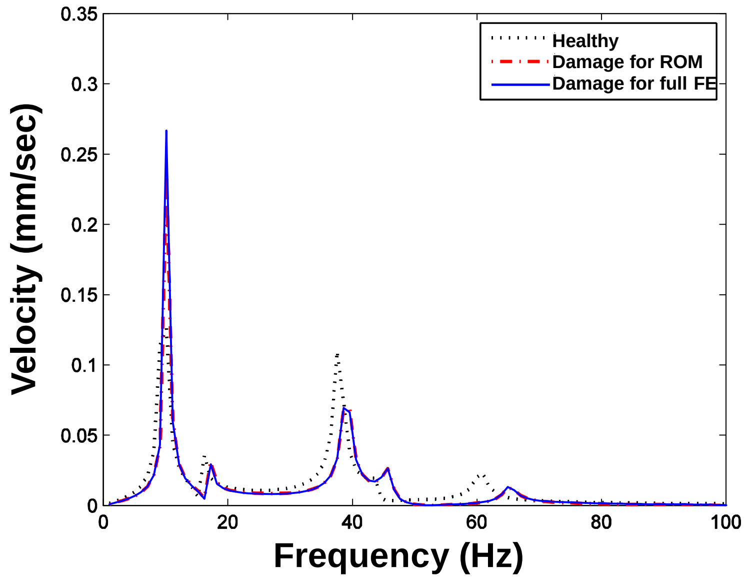
$\gamma = 0.03$ (structural damping)

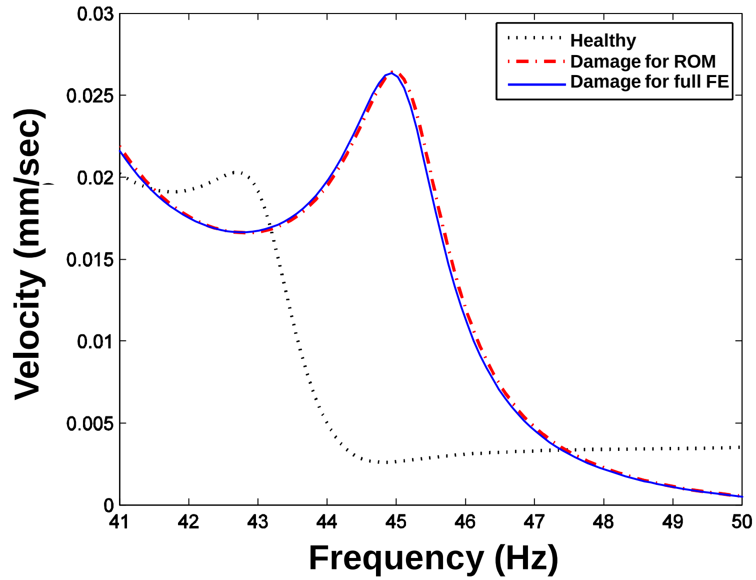
$u$ : physical coordinates

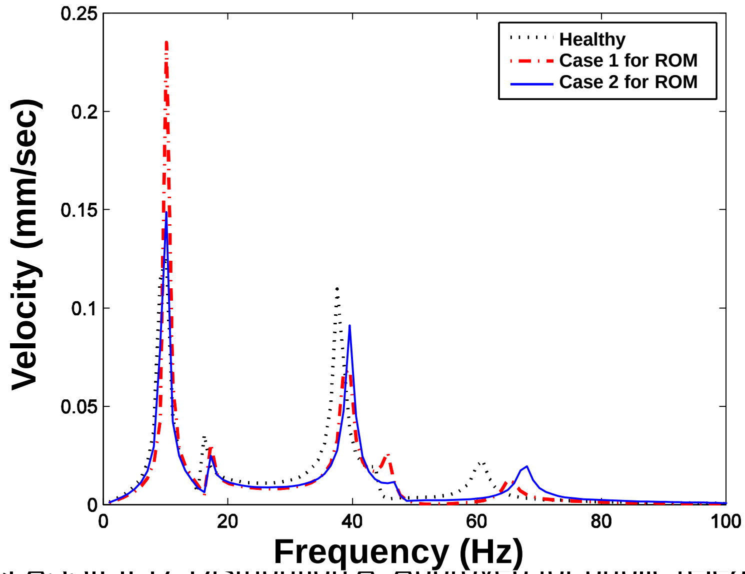
$q$ : modal coordinates

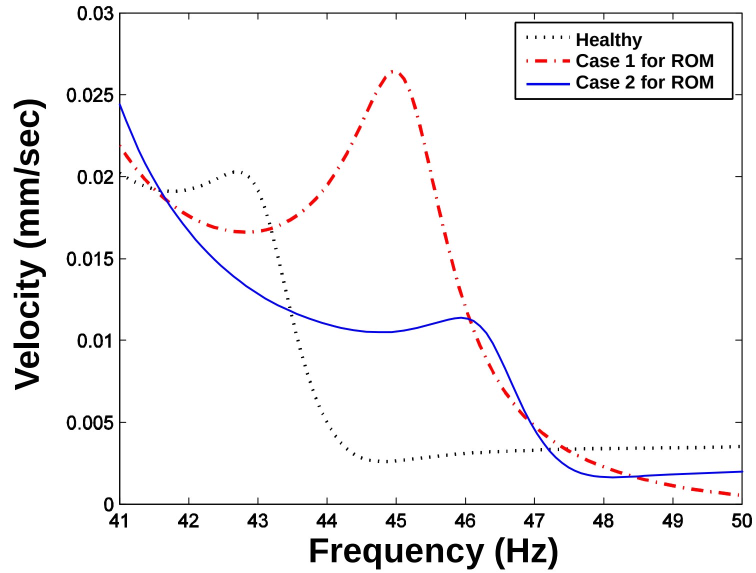
| Substructure | Thickness, Case 1 | Thickness, Case 2 |
|---|---|---|
| 1 | 0.4 mm → 0.473 mm | 0.4 mm → 0.435 mm |
| 6 | 0.4 mm → 0.422 mm | 0.4 mm → 0.491 mm |
| 7 | 0.4 mm → 0.493 mm | 0.4 mm → 0.481 mm |

# Results: L-Shape Plate: Forced Response



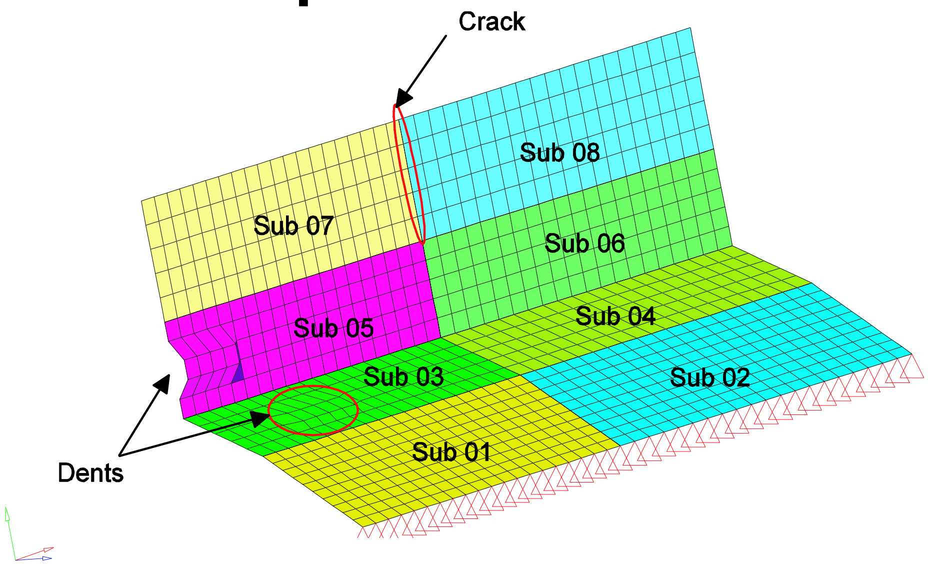



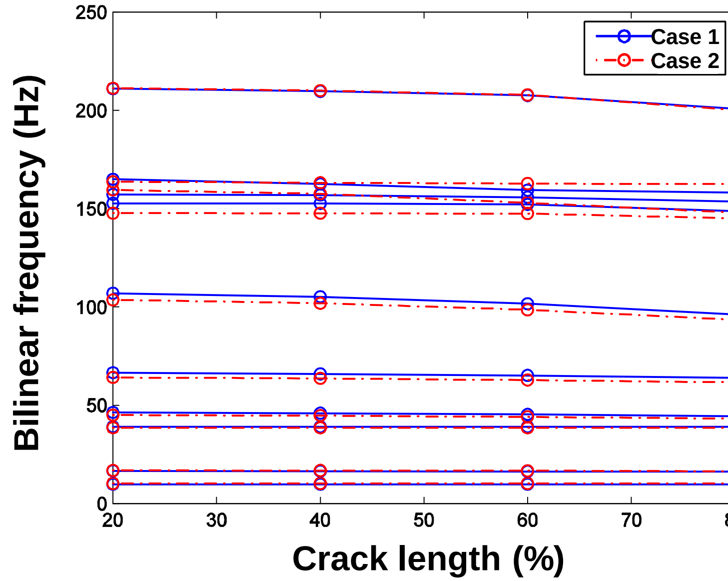



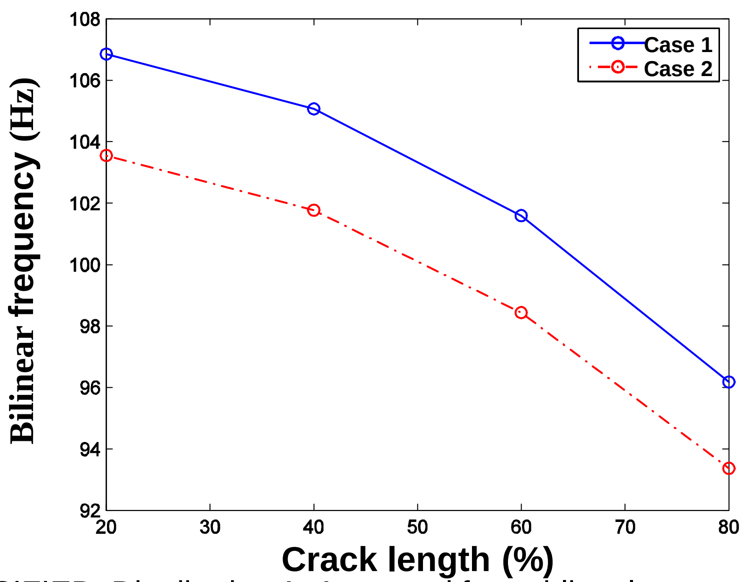



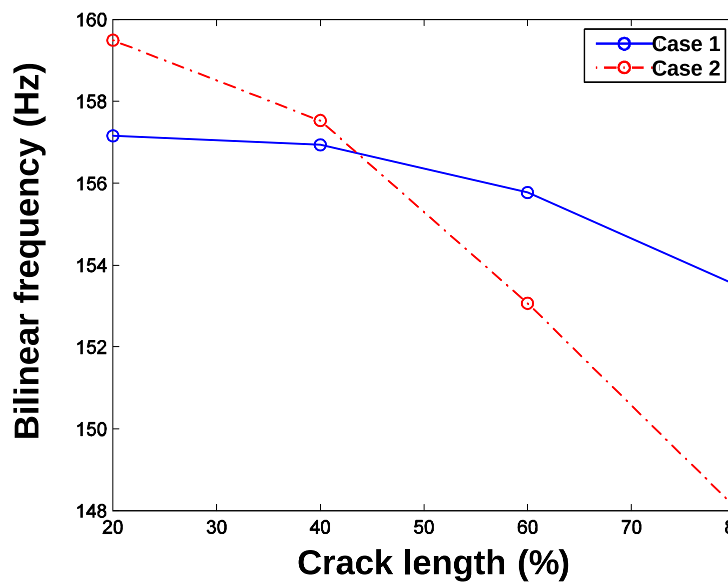
## Results: L-Shape Plate: Dents, Thickness Variations and Crack

### Free Response

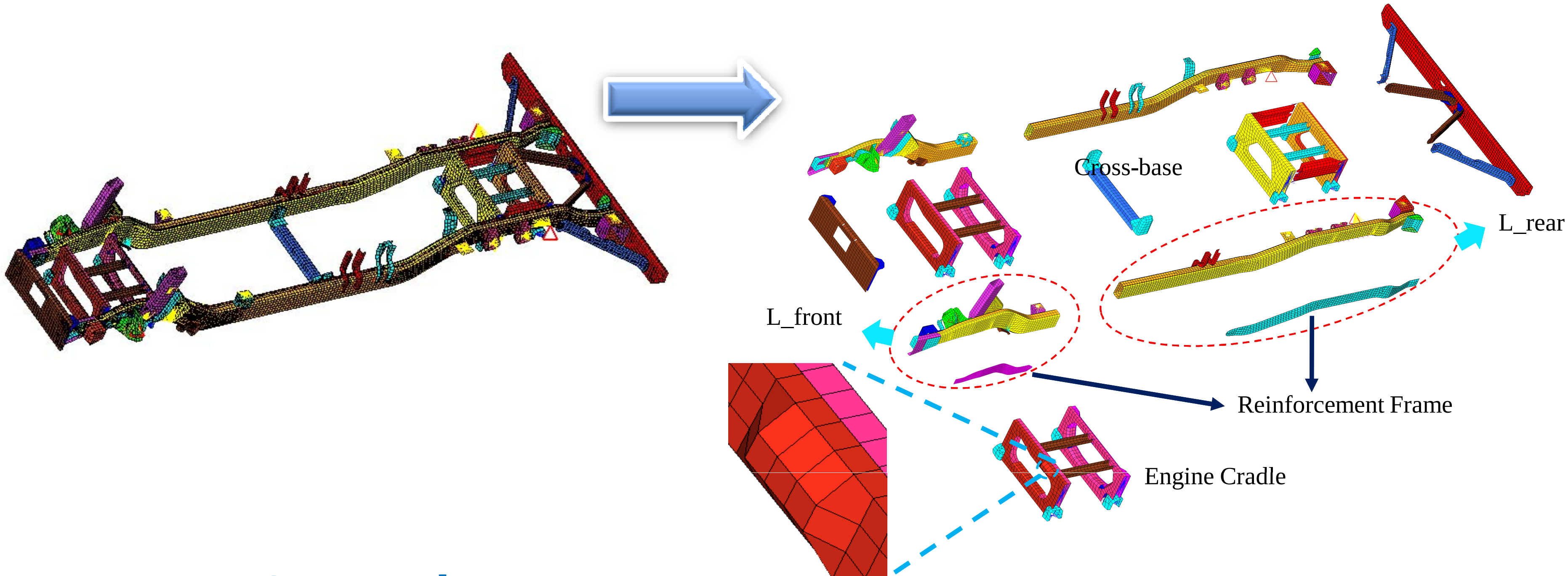


# Results: Vehicle Frame : Dents and Thickness Variations




## Damage Scenario

Each reinforcement frame has **thickness variation**

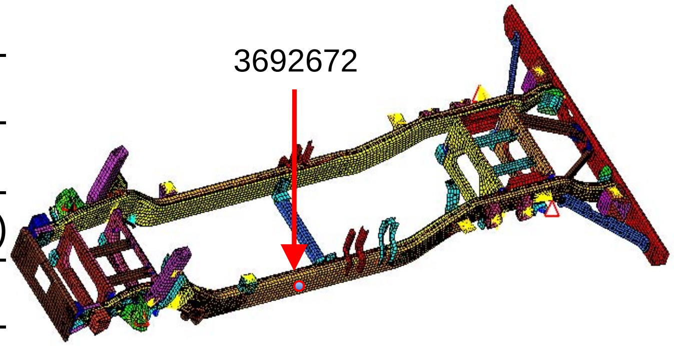
Engine cradle has a **dent**

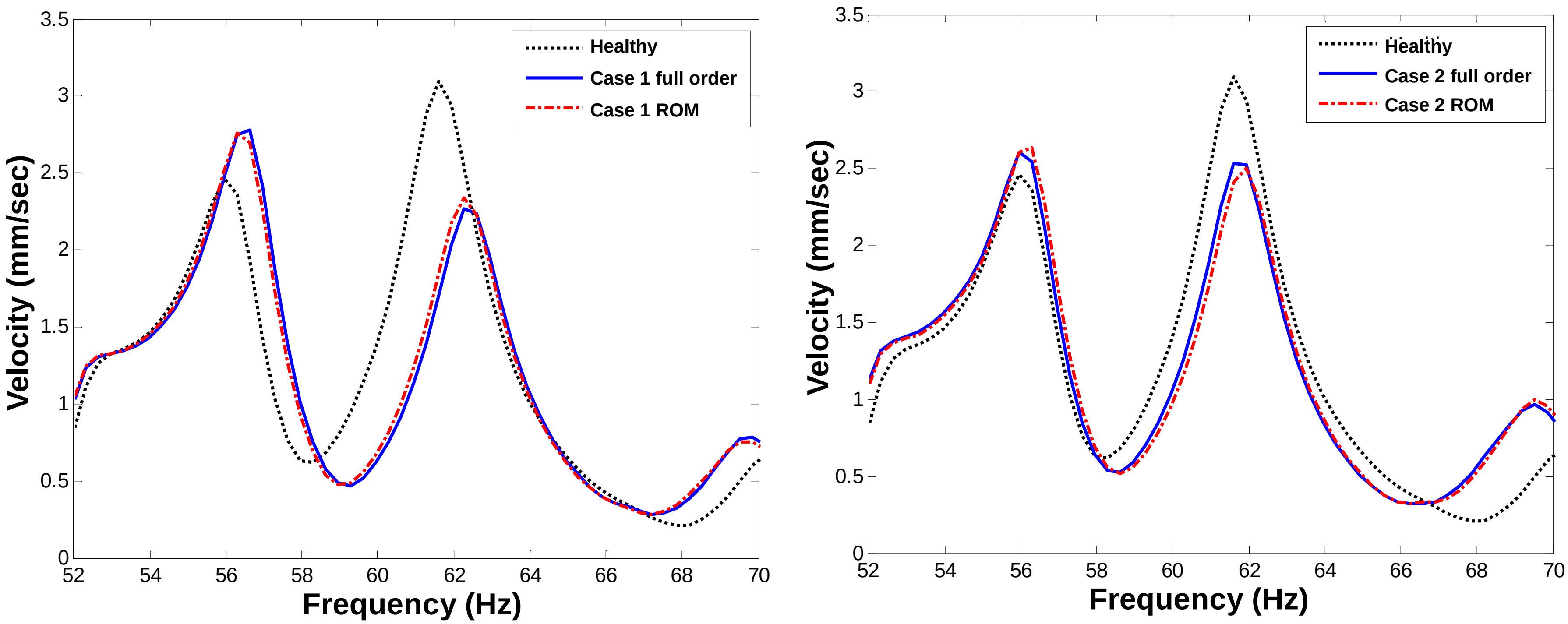
| Substructure | Thickness, case1 | Thickness, case2 |
|---|---|---|
| L_rear | 3.0378 mm → 4.6268 mm | 3.0378 mm → 5.5788 mm |
| L_front | 3.0378 mm → 5.3838 mm | 3.0378 mm → 4.0908 mm |

# Results: Vehicle Frame: Forced Response

Response point : 3692672

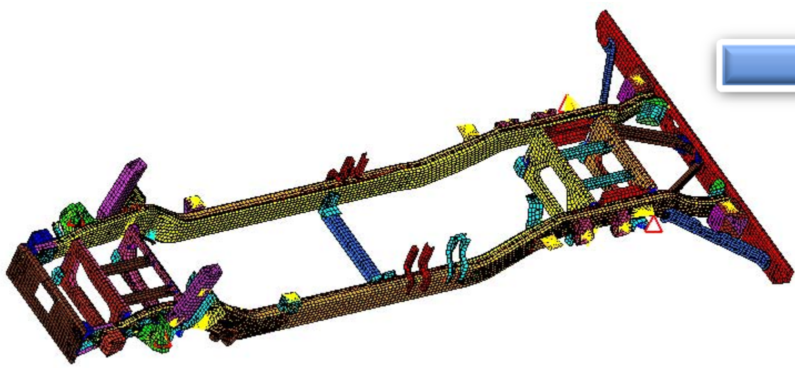
| | Full order model | | ROM |
|---|---|---|---|
| System DOF | 119808 | $\times\frac{1}{3}$ | 2420 |
| Initial Analysis time | 60125.216 (sec.) | → | 21955.959 (sec.) |
| Reanalysis time | 60125.216 (sec.) | → $\times\frac{1}{100}$ | 595.361 (sec.) |

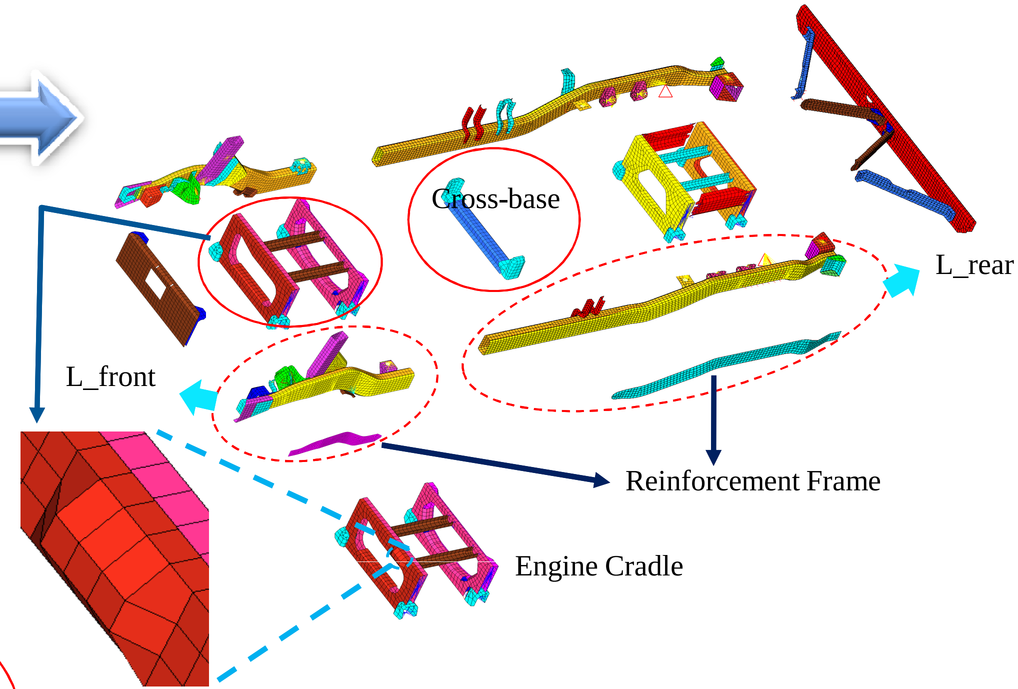


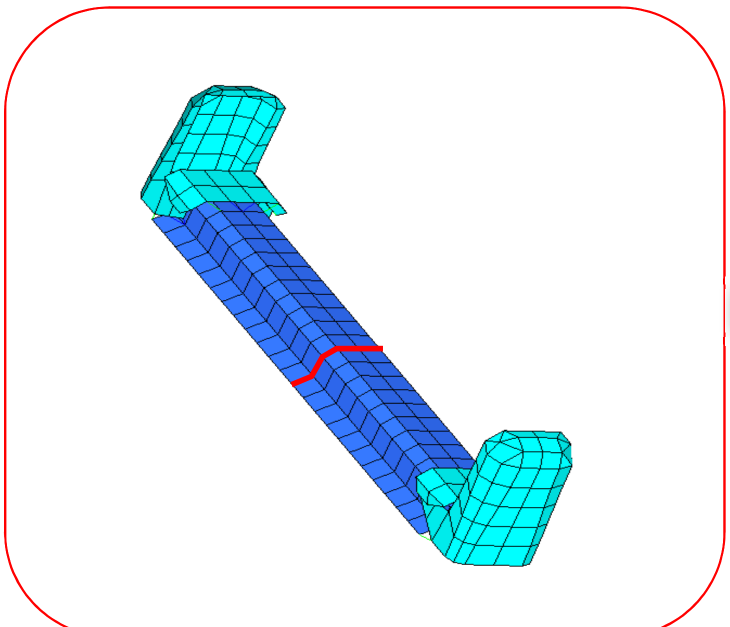



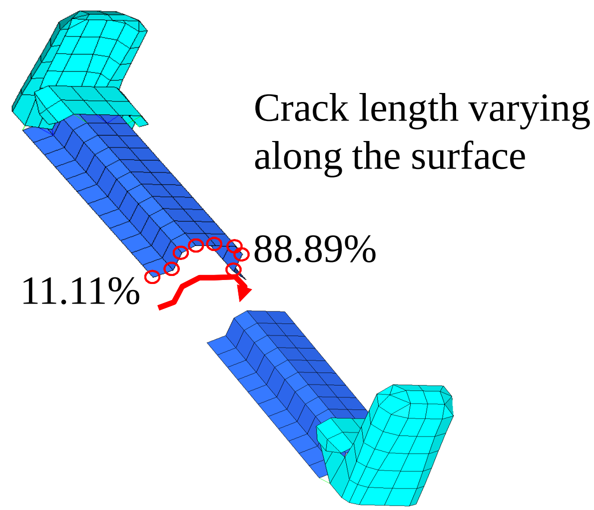

Forced response for cases 1 and 2

# Results: Vehicle Frame: Dents, Cracks and Thickness Variations

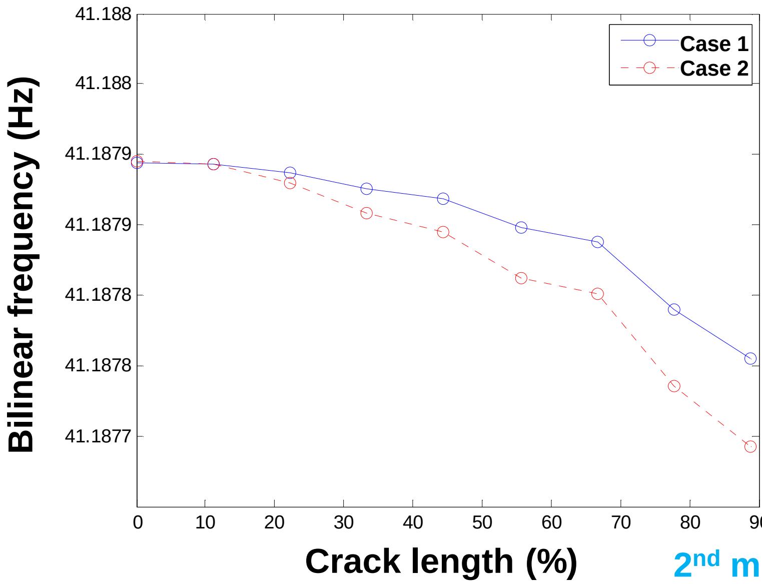




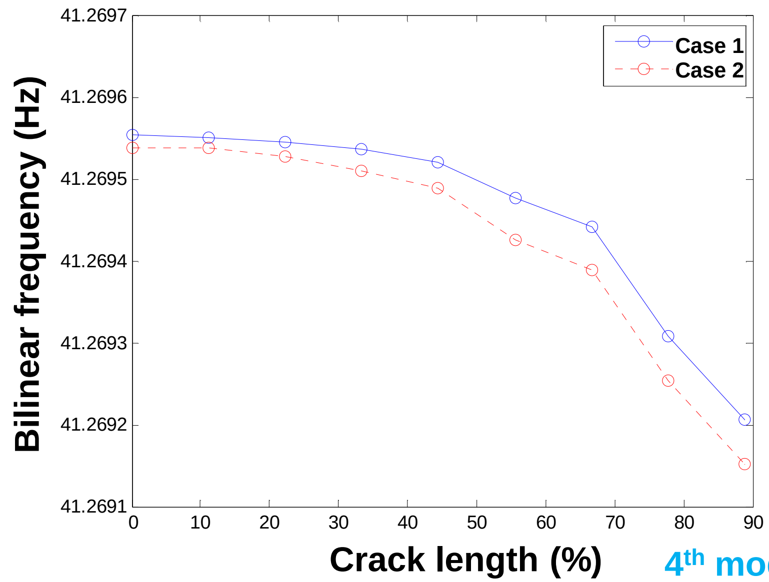





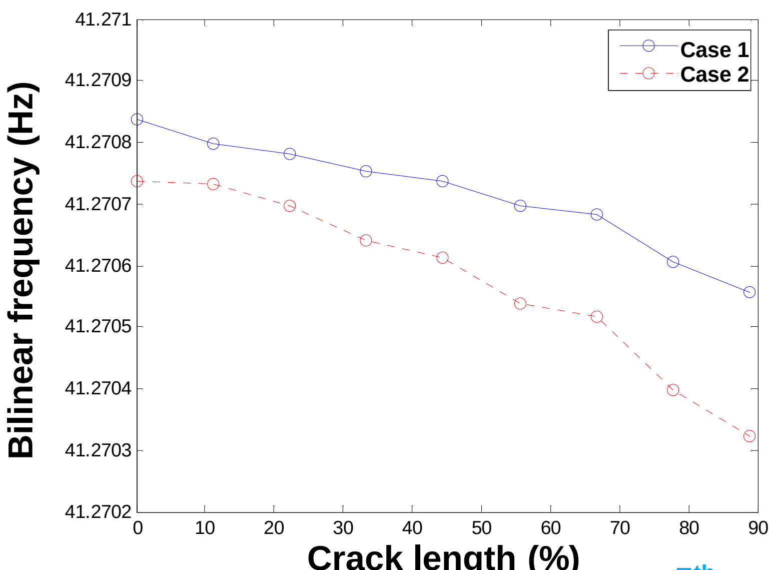
# Results: Vehicle Frame: Free Response



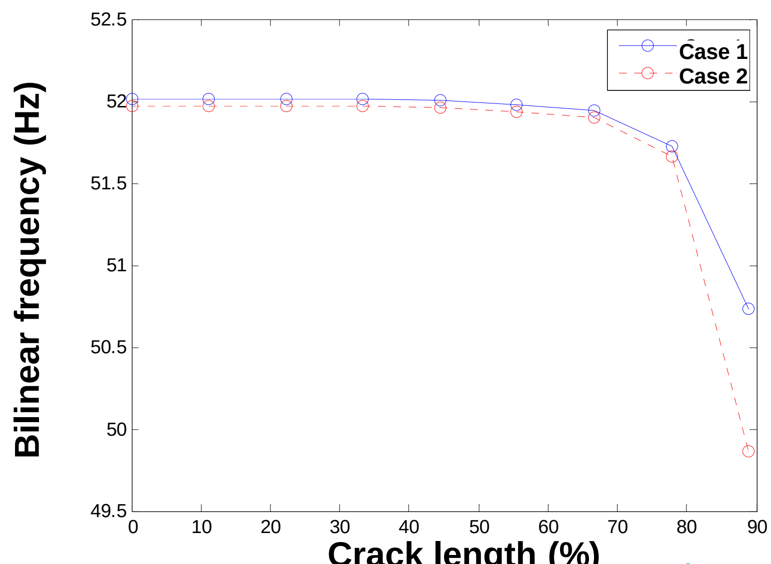

2nd mode




4th mode




5th mode



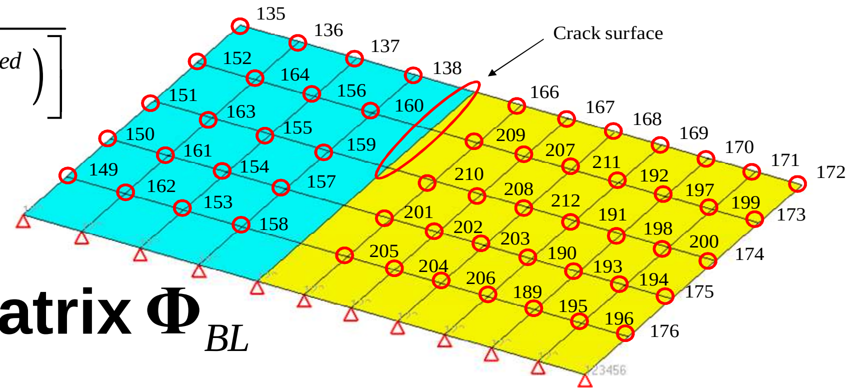

8th mode

# Bilinear Mode Approximation (BMA)

$$\mathbf{\Phi}_{BL,i}^{healthy} = \begin{bmatrix} \mathbf{\Phi}_i^{healthy} \\ \mathbf{\Phi}_i^{healthy} \end{bmatrix} \qquad \mathbf{\Phi}_{BL,i}^{damaged} = \begin{bmatrix} \mathbf{\Phi}_{open,i}^{ac} \\ \mathbf{\Phi}_{closed,i}^{ac} \end{bmatrix} \qquad \mathbf{M}_{BL} = \begin{bmatrix} \mathbf{M}_{CMS}^{open} & 0 \\ 0 & \mathbf{M}_{CMS}^{closed} \end{bmatrix}$$

# Modal assurance criterion (MAC): sensitive mode shapes

$$\mathbf{MAC}_{ij} = \frac{\left[ \left( \mathbf{\Phi}_{BL,i}^{healthy} \right)^T \mathbf{M}_{BL} \left( \mathbf{\Phi}_{BL,j}^{damaged} \right) \right]}{\left[ \left( \mathbf{\Phi}_{BL,i}^{healthy} \right)^T \mathbf{M}_{BL} \left( \mathbf{\Phi}_{BL,i}^{healthy} \right) \right] \left[ \left( \mathbf{\Phi}_{BL,j}^{damaged} \right)^T \mathbf{M}_{BL} \left( \mathbf{\Phi}_{BL,j}^{damaged} \right) \right]}$$




# Augmented bilinear (BL) modal matrix $\mathbf{\Phi}_{BL}$

$$\mathbf{\Phi}_{BL} = \begin{bmatrix} \mathbf{\Phi}^{healthy} & \mathbf{\Phi}_{open}^{ac1} & \mathbf{\Phi}_{open}^{ac2} \\ \mathbf{\Phi}^{healthy} & \mathbf{\Phi}_{closed}^{ac1} & \mathbf{\Phi}_{closed}^{ac2} \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}_{BL}^{healthy} & \mathbf{\Phi}_{BL}^{damaged1} & \mathbf{\Phi}_{BL}^{damaged2} \end{bmatrix}.$$

# Sensor Placement Algorithm for Cracked Structure

## General sensor placement algorithm: EIDV

- Effective independence distribution vector (EIDV) [Kammer, 1991; Penny et al., 1994]
- From the real modal matrix, the EIDV algorithm is executed.

## Problem

- The augmented **BL** modal matrix $\boldsymbol{\Phi}_{BL}$ can be linearly dependent

## Solution

- Use left singular vector U of $\boldsymbol{\Phi}_{BL}$ within the criteria to EIDV

$\boldsymbol{\Phi}_{BL} : (\mathrm{M}\times\mathrm{N})$ M : Number of candidate measurement DOF, N : Number of mode

1th to pth column

$$\boldsymbol{\Phi}_{BL} = \begin{bmatrix} \mathrm{M} & \mathrm{M} & \mathrm{M} \\ \mathbf{U}_1 & \mathrm{L} & \mathbf{U}_\mathrm{M} \\ \mathrm{M} & \mathrm{M} & \mathrm{M} \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 & 0 & \mathrm{L} & 0 \\ 0 & \sigma_2 & 0 & \mathrm{L} & 0 \\ 0 & 0 & \sigma_3 & \mathrm{O} & 0 \\ \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{O} & 0 \\ 0 & 0 & 0 & \mathrm{L} & \sigma_\mathrm{N} \\ 0 & 0 & 0 & \mathrm{L} & 0 \\ \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{O} & 0 \\ 0 & 0 & 0 & \mathrm{L} & 0 \end{bmatrix} \begin{bmatrix} \mathrm{L} & \mathbf{V}_1 & \mathrm{L} \\ \mathrm{M} & \mathrm{L} & \mathrm{M} \\ \mathrm{L} & \mathbf{V}_\mathrm{N} & \mathrm{L} \end{bmatrix}$$

Maximum singular value : $\sigma_1$
Minimum singular value : $\sigma_p$

## Criteria :

$\sigma_p$ is larger than 0.01% of the maximum singular value $\sigma_1$

Make $\boldsymbol{\Phi}_{SVD}$ for EIDV

$$\boldsymbol{\Phi}_{SVD} = \begin{bmatrix} \mathrm{M} & \mathrm{M} & \mathrm{M} \\ \mathbf{U}_1 & \mathrm{L} & \mathbf{U}_\mathrm{p} \\ \mathrm{M} & \mathrm{M} & \mathrm{M} \end{bmatrix}$$

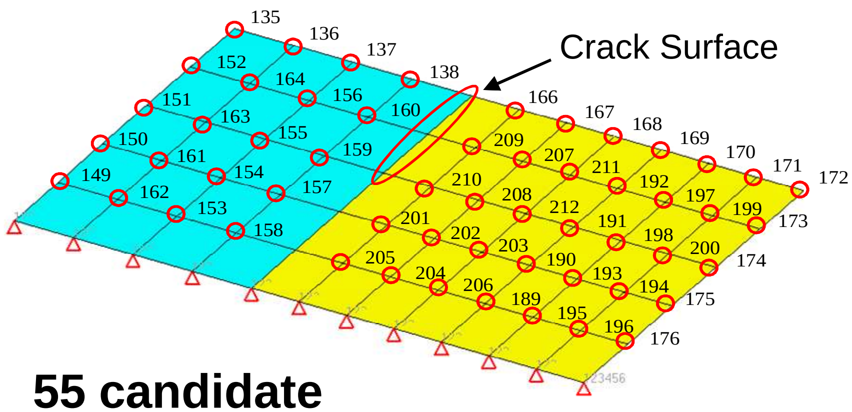
# Algorithm for modified EIDV with Left Singular Vector

- Calculate the mode shape for the healthy and damaged structures for open and closed cases in reduced order domain.
- Construct the *BL* modal matrix for the healthy and damaged structures.
- Find the sensitive mode shapes (and their frequencies) by using the generalized MAC matrix.
- Make bilinear augmented modal matrix $\mathbf{\Phi}_{BL}$ by the sensitive mode from the modified MAC matrix.
- Obtain the left singular vector U of $\mathbf{\Phi}_{BL}$ and make $\mathbf{\Phi}_{SVD}$ which is consist of left singular from $U_1$ to $U_p$ based on the criteria

$$\mathbf{\Phi}_{SVD} = \begin{bmatrix} \mathrm{M} & \mathrm{M} & \mathrm{M} \\ \mathbf{U}_1 & \mathrm{L} & \mathbf{U}_{\mathrm{p}} \\ \mathrm{M} & \mathrm{M} & \mathrm{M} \end{bmatrix}$$

- Calculate Fisher information matrix given by $\mathbf{A} = \mathbf{\Phi}_{SVD}{}^{T}\mathbf{\Phi}_{SVD}$.
- Calculate effective independence distribution vector (EIDV), the diagonal of $\mathbf{E} = \mathbf{\Phi}_{SVD}{}^{T}\mathbf{A}^{-1}\mathbf{\Phi}_{SVD}$.

# Example: Cracked plate

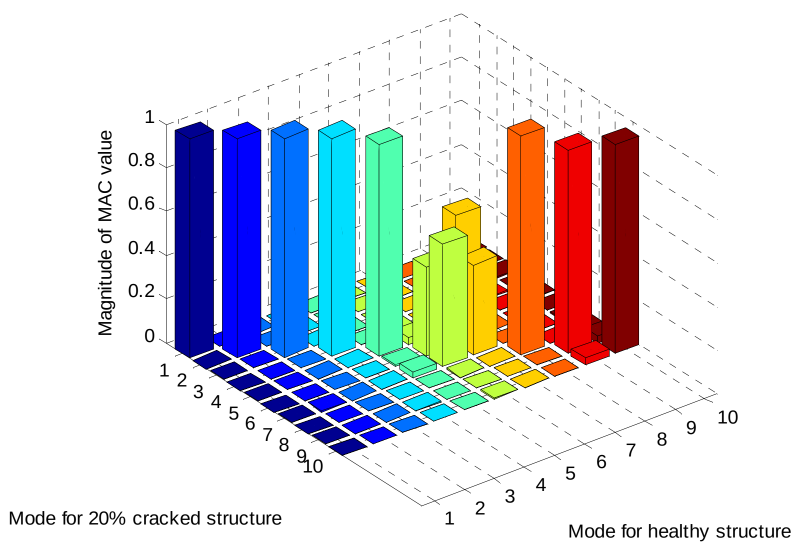



**55 candidate Measurement points**

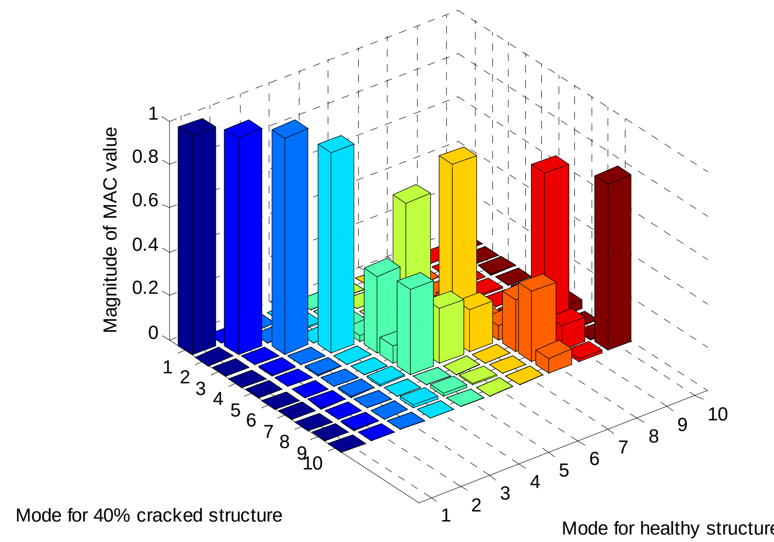
*1. Calculate the mode shapes for open and closed states at each crack length*

*2. Construct the BL modal matrix for the healthy and damaged structures*

*3. Find the sensitive mode shapes by using the generalized MAC matrix*

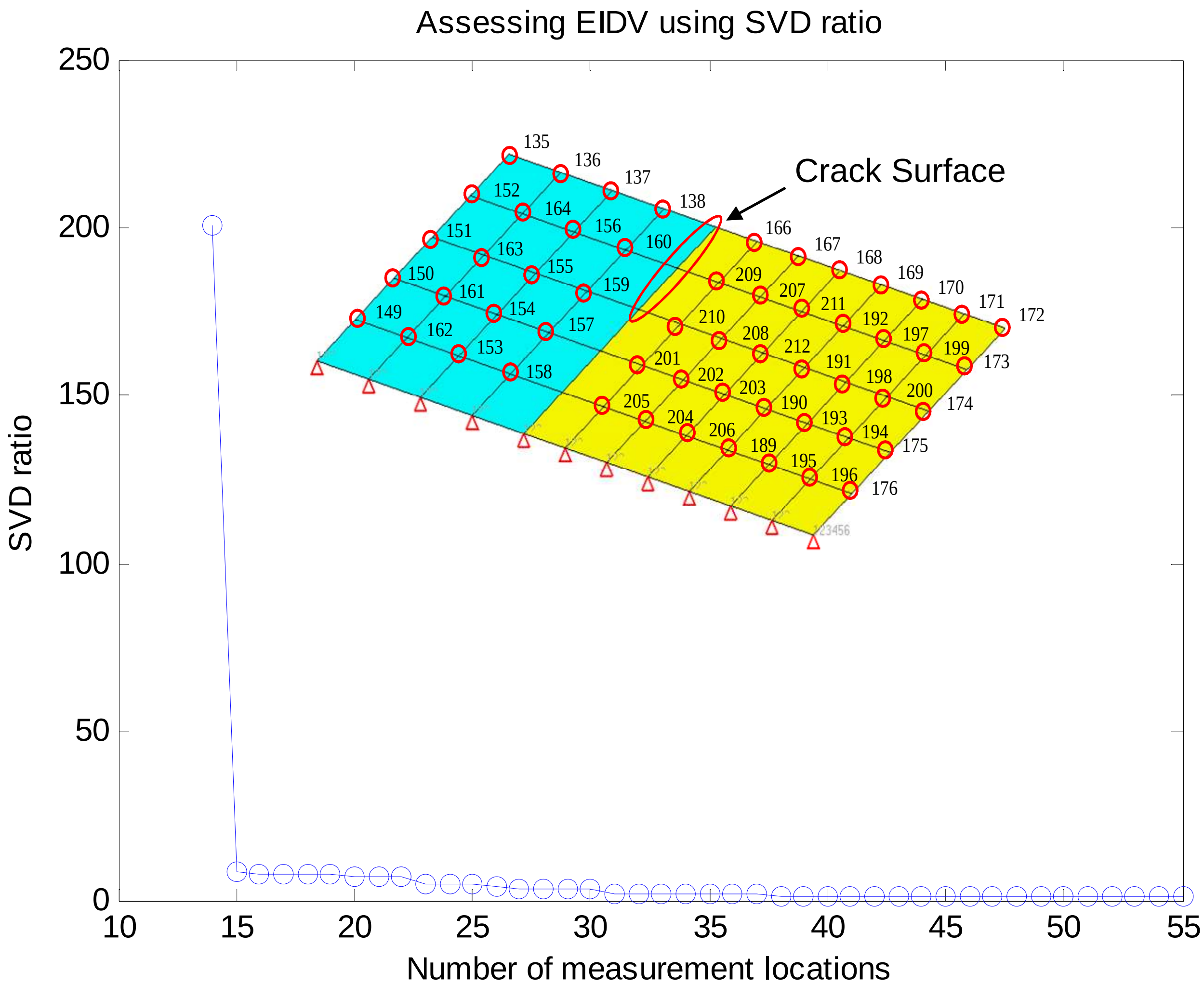


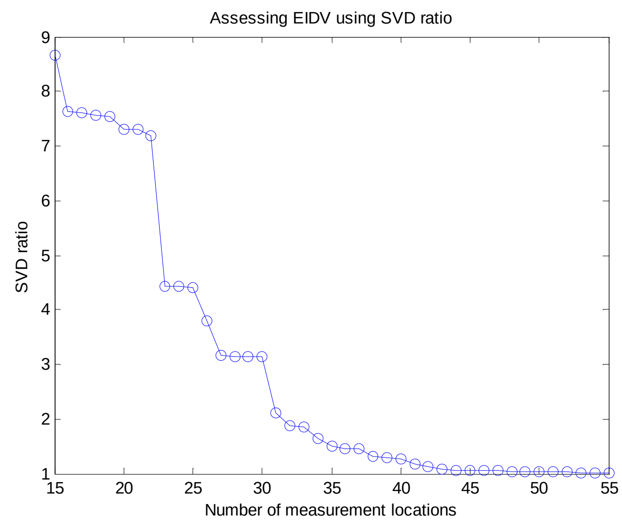

**20% cracked structure**



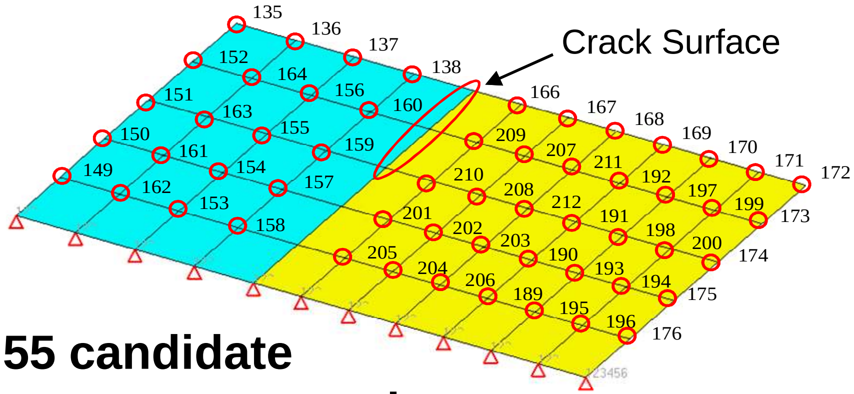

**40% cracked structure**

# SVD ratio versus number of measurement locations

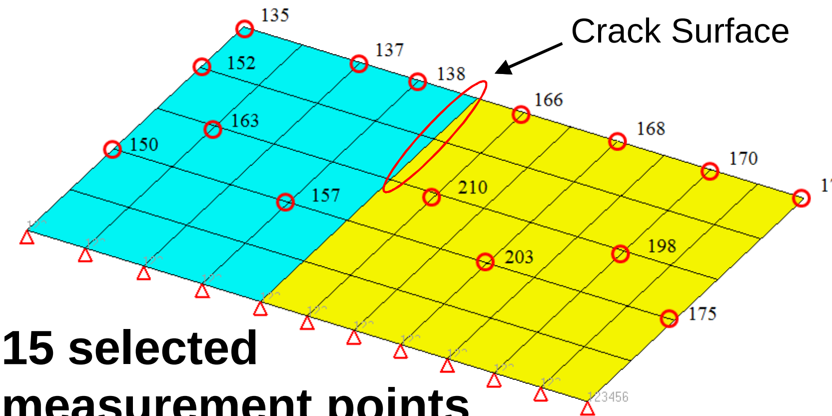


# Results: Measurement point selection




**55 candidate measurement points**

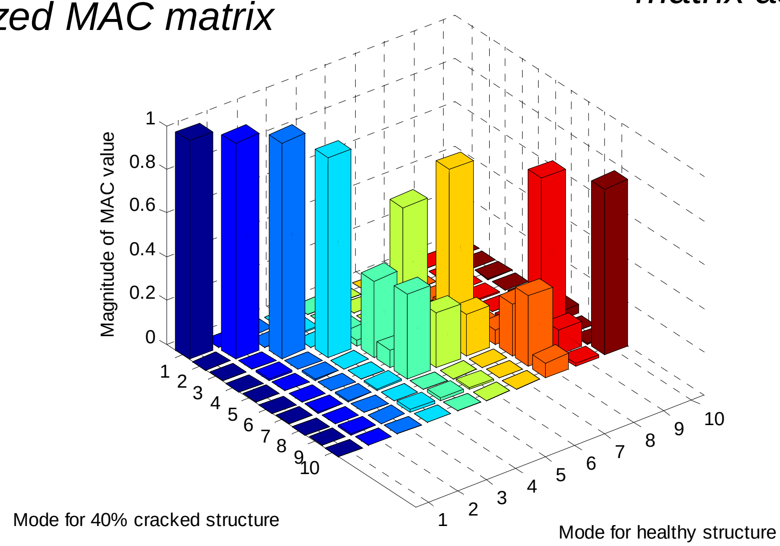



**15 selected measurement points**

*Find the sensitive mode shapes by using the generalized MAC matrix*

*Apply modified EIDV using bilinear modal matrix assembled with sensitive modes to select measurement points*

*(15 points in this example)*




*4th to 9th mode shapes are sensitive for healthy and 40% cracked structure*

# Summary

## Modeling and simulation of damaged structures

- Reduced-order models for dents, thickness changes, etc.
- Fast reanalysis methods
- Bilinear approximations for predicting nonlinear effects of cracks

## Sensor placement (measurement point selection) method

- Bilinear mode approximation (BMA)
- EIDV-based algorithm for point selection

## Future work

- Applications to **SHM of complex structures, joining/fastening**
- Applications to **design for reliability, observability**

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